

[RESEARCH]

Broadband RF Power Standard for 7 mm Coaxial Waveguide in the Frequency Range of 10 MHz-18 GHz - Evaluation of Uncertainty -

Takemi INOUE Keiko SATO

The evaluation of uncertainty of a broadband RF power calibration system were studied from 10 MHz to 18 GHz of 7 mm coaxial waveguide. The formula of uncertainty and expression of the factors was derived from measuring equation. They were summarized to three tables of the DC substitution coefficient, the power splitting ratio and the calibration factor. As a result, the standard uncertainty of the calibration system became 0.1~0.21 % for the frequency range from 10 MHz to 18 GHz. This is considered as the best uncertainty of the system. As an example, a power meter was calibrated and the standard uncertainty of its calibration factor became 0.15~0.65 % in the full frequency range.

§1 Introduction

With the recent development of radio frequency (RF) techniques, various RF instruments have been used and the frequency range are coming higher and broader. To guarantee the reliability and accuracy, it is necessary to establish traceability based on a standard and to calibrate them. The basic quantities such as power, impedance, attenuation and so on are basically important for their calibration. Above all RF power is important and we have already developed power standards of a 14 mm coaxial waveguide in the point frequency of 1 and 3 GHz. Broadband RF power standards are now under development. At present, especially, frequency range from 10 MHz to 18 GHz is widely used and kinds of instruments are commercially available. Here, we have studied and developed a broadband RF power standard of 7 mm coaxial waveguide in those frequency range. It consists of a calibration system for a power meter with a APC 7 connector based on a broadband calorimeter. The design and fabrication of the system is presented in another report in this bulletin¹⁾. Here, the evaluation of the measurement uncertainty is described.

§2 Principle of the calibration

Fig.1 shows the calibration system of a broadband RF power meter. This system uses a 7mm coaxial waveguide and a APC 7 connector. The principle of the power measurement is based on a calorimetric method using an isothermal control²⁾. At first the constant current source supplies appropriate DC current to the

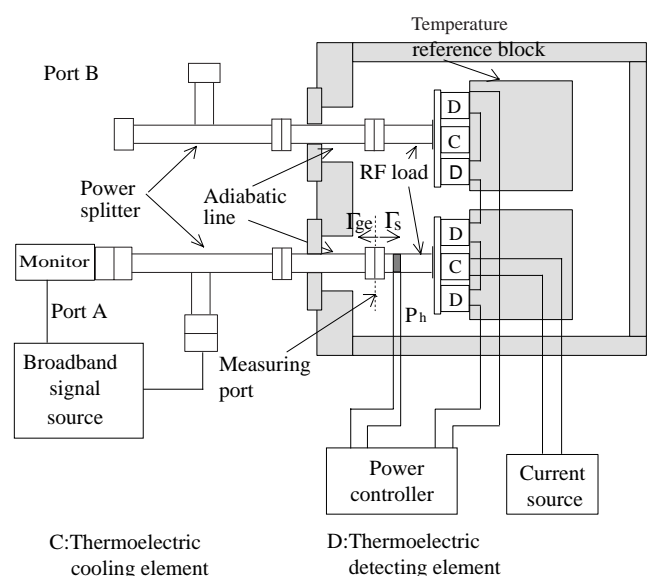


Fig. 1 The coaxial broadband power calibration system

KEY WORDS : Radio frequency, power, standard, cakibration, uncertainty

thermoelectric cooling element and the RF load is refrigerated. Then the detected voltage from the thermoelectric detecting elements are fed back to the built-in heater of the RF load. As the result, the temperature of the RF load comes to that of the temperature reference block. Under this isothermal control, the heater bias power change comes to the absorbed RF power, while RF turned on and off. The signal source is composed of a broadband synthesizer, a power splitter and a monitor power meter.

The calibration is performed by computer programs and the procedure is composed of the following three steps¹⁾.

- ! Substitution coefficient k : Substitution coefficient of the RF load and heater power
- " Power splitting ratio C_d : The ratio of the calorimetric measured RF power and monitor power
- # Calibration factor K of a device under test (DUT)

In the calibration, the outer jacket, the cooling/detection part and the RF load are removed and a device under test (DUT) is connected to the measuring port. The measurement of the step " and # are carried out for the frequency range prescribed. The calibration factor of DUT from these measurement is expressed as¹⁾

$$K = \frac{P_u}{C_d P_{ms} \eta_l \eta_a |1 - \Gamma_g \Gamma_u|^2} \quad (1)$$

Where, P_u is the measured power by a DUT. C_d is the power splitting ratio that is measured RF power P_{rf} divided by the monitor power P_{ms} . P_{ms} is the monitor power for the calibration of a DUT. Γ_g is the reflection coefficient of the signal source and Γ_u is that of the DUT. η_a and η_l are the power transmission efficiency of an adapter and an extension line, respectively.

If $|1 - \Gamma_g \Gamma_u|^2$ in Eq. (1) is evaluated separately as independent uncertainty factor, the measurement equation comes

$$K = \frac{P_u}{C_d P_{ms} \eta_l \eta_a} \quad (2)$$

On the other hand, C_d is determined by the following equation¹⁾.

$$C_d = \frac{k \left(1 - \frac{q\alpha}{1 - \alpha}\right) P_h}{P_{ms} (1 - |\Gamma_l|^2) |1 - \Gamma_g \Gamma_l|^2} \quad (3)$$

Where, k is the substitution coefficient of RF and DC. It is defined as the ratio of DC power input to RF load and calorimetric measured power. P_h is the RF power measured by the calorimeter. P_{ms} is the monitor power measured with calorimetric RF power measurement. Γ_l is the reflection coefficient of the RF load. q is the ratio of heat transferred to the RF load to the heat generated due to the loss of the adiabatic line. α is the rate of the attenuation by the adiabatic line.

The substitution coefficient k is determined by calorimetric measurement supplying positive and negative current into the RF load through the input port of the calorimeter. k is given by the following equation¹⁾.

$$k = \frac{k_+ + k_-}{2} \quad (4)$$

k_+ and k_- is calculated as in the following.

$$k_+ = \frac{RI_{dc+}^2}{P_{h+}} \quad (5)$$

$$k_- = \frac{RI_{dc-}^2}{P_{h-}} \quad (6)$$

Where, k_+ and k_- is the substitution coefficient for positive and negative DC current, respectively. R is resistance of the RF load. I_{dc+} and I_{dc-} is the applied current to the RF load, respectively. P_{h+} and P_{h-} is the calorimetric measured heater power for positive and negative current, respectively.

§3 The evaluation equation of the uncertainty

3.1 Uncertainty of calibration factor measurement of a device under test

The evaluation of the uncertainty has been made based on the ISO Guide³⁾. The combined standard uncertainty of the calibration factor measurement of a DUT is derived from the equation (1) and expressed by

$$u_c^2(K) = \left(\frac{\partial f}{\partial C_d}\right)^2 u^2(C_d) + \left(\frac{\partial f}{\partial P_u}\right)^2 u^2(P_u) + \left(\frac{\partial f}{\partial P_{mu}}\right)^2 u^2(P_{mu}) + \left(\frac{\partial f}{\partial \eta_l}\right)^2 u^2(\eta_l) + \left(\frac{\partial f}{\partial \eta_a}\right)^2 u^2(\eta_a) + \epsilon_{smu}^2 + s^2(K). \quad (7)$$

Where, $K=f(C_d, P_u, P_{mu}, \eta_l, \eta_a)$ and the definition of this function is used only in this section. ϵ_{smu} is the uncertainty due to the source mismatch for calorimetric measurement. $s(K)$ is the standard deviation of measured K value.

Among these, the parameter which depends on the frequency are : C_d, η_l and η_a .

Each sensitivity coefficient is as follows.

$$\frac{\partial f}{\partial P_u} = \frac{K}{P_u} \quad (8) \quad \frac{\partial f}{\partial C_d} = \frac{K}{C_d} \quad (9) \quad \frac{\partial f}{\partial P_{mu}} = \frac{K}{P_{mu}} \quad (10)$$

$$\frac{\partial f}{\partial P_{mu}} = \frac{K}{P_{mu}} \quad (11) \quad \text{and} \quad \frac{\partial f}{\partial \eta_a} = \frac{K}{\eta_a} \quad (12)$$

3.2 Uncertainty of the power splitting ratio

The uncertainty of the power splitting ratio C_d in the calorimetric RF power measurement is shown as follows from the equation (3).

$$u_c^2(C_d) = \left(\frac{\partial f}{\partial k}\right)^2 u^2(k) + \left(\frac{\partial f}{\partial q}\right)^2 u^2(q) + \left(\frac{\partial f}{\partial \alpha}\right)^2 u^2(\alpha) + \left(\frac{\partial f}{\partial |\Gamma_l|}\right)^2 u^2(|\Gamma_l|) + \left(\frac{\partial f}{\partial P_h}\right)^2 u^2(P_h) + \left(\frac{\partial f}{\partial P_{ms}}\right)^2 u^2(P_{ms}) + \epsilon_{sms}^2 + s^2(C_d). \quad (13)$$

Where, $C_d=f(k, q, \alpha, |\Gamma_l|, P_h, P_{ms})$ and the definition of this function is used only in this section. ϵ_{sms} is the uncertainty due to the source mismatch for calorimetric measurement. $s(C_d)$ is the standard deviation of measured C_d value.

Among these, the parameter which depends on the frequency are : α and $|\Gamma_l|$.

Each sensitivity coefficient is as follows.

$$\frac{\partial f}{\partial k} = \frac{C_d}{k} \quad (14) \quad \frac{\partial f}{\partial q} = \frac{\alpha C_d}{1 - \alpha - q\alpha} \quad (15)$$

$$\frac{\partial f}{\partial \alpha} = \frac{qC_d}{(1 - \alpha - q\alpha)(1 - \alpha)} \quad (16) \quad \frac{\partial f}{\partial |\Gamma_l|} = \frac{2C_d|\Gamma_l|}{1 - |\Gamma_l|^2} \quad (17)$$

$$\frac{\partial f}{\partial P_h} = \frac{C_d}{P_h} \quad (18) \quad \frac{\partial f}{\partial P_{ms}} = \frac{C_d}{P_{ms}} \quad (19)$$

3.3 Uncertainty of the substitution coefficient

The uncertainty of the substitution coefficient k is expressed as follows from equations (4)~(6).

$$u_c^2(k) = \left(\frac{\partial f}{\partial k_+}\right)^2 u^2(k_+) + \left(\frac{\partial f}{\partial k_-}\right)^2 u^2(k_-) \quad (20)$$

where, $k = f(k_+, k_-)$.

$$u_c^2(k_+) = \left(\frac{\partial f_+}{\partial R}\right)^2 u^2(R) + \left(\frac{\partial f_+}{\partial I_{dc+}}\right)^2 u^2(I_{dc+}) + \left(\frac{\partial f_+}{\partial P_{h+}}\right)^2 u^2(P_{h+}) + s^2(k_+) \quad (21)$$

where, $k_+ = f_+(R, I_{dc+}, P_{h+})$.

$$u_c^2(k_-) = \left(\frac{\partial f_-}{\partial R}\right)^2 u^2(R) + \left(\frac{\partial f_-}{\partial I_{dc-}}\right)^2 u^2(I_{dc-}) + \left(\frac{\partial f_-}{\partial P_{h-}}\right)^2 u^2(P_{h-}) + s^2(k_-) \quad (22)$$

where, $k_- = f_-(R, I_{dc-}, P_{h-})$.

The definitions of these function f, f_+ and f_- is used only in this section. $s(k_+)$ and $s(k_-)$ are the standard deviation for positive and negative DC current, respectively.

Each sensitivity coefficient is as follows.

$$\frac{\partial f}{\partial k_+} = \frac{1}{2} \quad (23) \quad \frac{\partial f}{\partial k_-} = \frac{1}{2} \quad (24)$$

$$\frac{\partial f_+}{\partial R} = \frac{k_+}{R} \quad (25) \quad \frac{\partial f_+}{\partial I_{dc+}} = \frac{2k_+}{I_{dc+}} \quad (26)$$

$$\frac{\partial f_+}{\partial I_{h+}} = -\frac{k_+}{P_{h+}} \quad (27) \quad \frac{\partial f_-}{\partial R} = \frac{k_-}{R} \quad (28)$$

$$\frac{\partial f_-}{\partial I_{dc-}} = \frac{2k_-}{I_{dc-}} \quad (29) \quad \frac{\partial f_-}{\partial I_{h-}} = \frac{k_-}{P_{h-}} \quad (30)$$

§4 Evaluation of uncertainty factor

4.1 Uncertainty factor of calibration of DUT

- (1) Power splitting ratio: $u(C_d)$

It is gotten from the result with 4.2 paragraph.

- (2) RF power measurement by a DUT: $u(P_u)$

This is considered as 1 digit of the indicator of the DUT.

- (3) RF power measurement by the monitor power meter : $u(P_{mu})$

This is considered as 1 digit of the monitor power meter.

- (4) Efficiency of an extension line: $u(\eta_l)$

This is the uncertainty of the efficiency of the line used for the extension of the measuring port and when not using, it is zero.

- (5) Efficiency of an adapter : $u(\eta_a)$

The measuring port of this calorimeter is APC 7 connector but it is necessary to use a conversion adapter for different type connector like a N type connector. It is with the efficiency of this adapter and when not using, it is zero.

- (6) Source mismatch: ϵ_{smu}

It is due to the multiple reflection between the signal source and a DUT, and comes $2|\Gamma_g \Gamma_l|$ as for the maximum. Assuming U distribution of the probability function, the standard uncertainty leads to the value divided by $\sqrt{2}$.

- (7) Standard deviation of the measured value of K : $s(K)$

4.2 Uncertainty factor of the power splitting ratio

- (1) Substitution coefficient: $u_c(k)$

It is given by the combined standard uncertainty of the substitution coefficient with the paragraph 4.3.

- (2) Excess heating by the adiabatic line: $u(q)$

This was evaluated by inserting a thin gold foil between the measuring port and the RF load, and performing calorimetric measurement for AC or RF input⁴⁾. As the AC, 1 kHz was chosen because of minimum uncertainty of AC measurement by the DVM. As a result, $q_{ac}=0.1941$ was gotten.

On the other hand, an influence of the excess heating with RF is shown in the term $q\alpha/(1-\alpha)$ of the equation (3). We can know the value of q from RF loss ratio measurement described next and calorimetric RF power measurements. From these experiments, q was obtained as 0.1367 in the frequency range. As a result, q was determined as 0.165 by averaging q for AC and RF input. The uncertainty is given as the difference divided by $2\sqrt{3}$ assuming a uniform distribution of the probability function, and it comes to 0.017.

- (3) Attenuation of the adiabatic line: $u(\alpha)$

The RF attenuation ratio α of the serial two adiabatic lines which are used for the calorimeter input part are measured in all the frequency ranges with a thermo-type power meter. The fitting curve by the 5 th polynomial data are used for the correction in the measurement. Therefore, linearity of the power meter and the difference between the approximation and measured data become the uncertainty factor.

- (4) Source mismatch: ϵ_{sms}

It is due to the multiple reflection between the signal source and the RF load, and comes $2|\Gamma_g \Gamma_l|$ as for the maximum. The standard uncertainty is the value divided by $\sqrt{2}$ same as 4.1(6).

4.3 Uncertainty factor of the substitution coefficient

The uncertainty of substitution coefficient k has the factors of DC resistance of the RF load, DC current setting and the heater power measurement and so on, as shown in the equation (21) and (22). This is explained by the fixed parameter with the following paragraph.

§5 Example of uncertainty evaluation

As an example, a calibration of a power meter with N type connector with coaxial 7 mm waveguide was carried out for 39 points from 10 MHz up to 18 GHz. The DUT was connected to the measuring port with an N to APC 7 conversion adapter. The data processing in the amount of the calibration factor went with the

Table 1 Parameter table (frequency independent)

Associated measurand	Input quantity	Input estimate	Uncertainty	Unit	Note
k	k+	1	0.00015	mA	of 5 times measurement
	k-	1	0.0001		of 5 times measurement
	R	49.905	0.002		Resistance measurement
	Idc+	14.14	0.0092		Current source
	Ph+	10	0.00068		DC power measurement
	Idc-	14.14	0.0092		Current source
Cd	Ph-	10	0.00068	mW	DC power measurement
	q	0.167	0.059		

Table 2 Uncertainty budget of the substitution coefficient

Input quantity	Input estimate Xi	Unit	Source of uncertainty	Type	Distribution	Sensitivity Ci	Standard uncertainty u(Xi)	Ci u(Xi) =ui(Xi)
DC resistance of RF load	49.875	Ω	DC resistance measurement	B	Normal	0.0200	2.00E-0	4.00E-05
Plus current (Idc+)	0.01414	A	Current measurement	B	Normal	70.5233	9.24E-06	6.52E-04
Power for Ih+ (Ph+)	0.01	W	Power measurement	B	Normal	-49.8599	6.80E-07	3.39E-05
k+	1.0000	W	Standard deviation of measurement	A	t	0.5	6.71E-05	3.35E-05
Minus current (Idc-)	0.01414	A	Current measurement	B	Normal	70.5233	9.24E-06	6.52E-04
Power for Ih- (Ph-)	0.01	W	Power measurement	B	Normal	-49.8599	6.80E-07	3.39E-05
k+	1.0000	W	Standard deviation of measurement	A	t	0.5	4.47E-05	2.24E-05

$$\text{Measurement equation } k = \frac{R}{2} \left(\frac{Idc_+^2}{Ph_+} + \frac{Idc_-^2}{Ph_-} \right)$$

Combined standard uncertainty

0.0009

spreadsheet software.

At first, the constant parameter which doesn't depend on the frequency to use for the evaluation of the uncertainty of k and C_d , is shown in **Table 1**. From these parameters, the uncertainty of the k measurement by the calorimeter was derived as shown in **Table 2** and the combined standard uncertainty became 0.0009.

In the C_d measurement, the parameters which depend on the frequency include α , P_h , P_{ms} , Γ_s and Γ_g as shown in equation (3). In the DUT measurement, they are P_u , P_{mu} , η_a , Γ_u and Γ_g as shown in equation (1). These data were tabulated at all frequencies with the input estimation quantity and the standard uncertainty.

This table is not shown in this report because of the big size.

The evaluation of the C_d measurement were performed for all frequency range. An example of uncertainty budget for 18GHz is shown in **Table 3**. In this table, the sign "U" means U-distribution of which probability function is proportional to square of cosine of the phase difference between Γ_g and Γ_s . The source mismatch which isn't contained in the measurement equation was considered as an independent factor. This factor is larger than the others. The combined standard uncertainty of the C_d measurement became 0.0021 at 18 GHz.

Table 3 Uncertainty budget of the power splitting ratio

f= 18 GHz									
Input quantity	Input estimate Xi	Unit	Source of uncertainty	Type	Distribution	Sensitivity Ci	Standard uncertainty u(Xi)	Ci u(Xi) =ui(Ku)	
RF power (Ph)	10	mW	Rf power measurement (1σ)	0.00068	B	Normal	0.1062	0.0007	7.22E-05
Monitor power (Pms)	10	mW	Resolution (±δ Xi)	0.001	B	Uniform	-0.1062	0.0003	3.06E-05
Reflection of RF load (Γs)	0.02	none	Reflection measurement (1σ)	0.003	B	Normal	0.0425	0.0030	1.27E-04
			Resolution (±δ Xi)	0.0001	B	Uniform	0.0425	0.0000	1.23E-06
Substitution coefficient (k)	1.0000	none	Measurement (1σ)	0.0009	B	Normal	1.0617	0.0009	9.82E-04
Excess heating ratio (q)	0.017	none	Measurement (1σ)	0.0017	B	Uniform	0.0374	0.0098	3.67E-04
Attenuation of adiabatic line	0.034	none	Approximation (1σ)	0.0003	B	t	0.0188	0.0003	4.90E-06
			Resolution (±δ Xi)	0.0001	B	Uniform	1.0188	0.0000	5.42E-07
Measured Cd	1.0617	none	Measurement (σn-1)	0.0012	A	t	1	0.0005	4.97E-04
Source mismatch		none	Γg	0.0058	B	U	1.0617	0.0016	1.74E-03
			Γs	0.02					

Measurement equation $C_d = \frac{k}{1-|\Gamma_s|^2} \left(1 - \frac{q\alpha}{1-\alpha} \right) \frac{P_h}{P_{ms}} \left| 1 - \Gamma_g \Gamma_s \right|^2$ Combined standard uncertainty 0.0021

Table 4 Uncertainty budget of the calibration factor

f= 18 GHz									
Input Quantity	Input estimate Xi	Unit	Source of uncertainty	Type	Distribution	Sensitivity Ci	Standard uncertainty u(Xi)	Ci u(Xi) =ui(Ku)	
Power splitting ratio (Cd)	1.0617	none	Cd measurement (1σ)	0.0021	B	Normal	-0.8730	0.0021	1.83E-03
RF power (Pu)	10	mW	Resolution (±δXi)	0.001	B	Uniform	0.0927	0.0003	2.68E-05
Monitor power (Pmu)	10	mW	Resolution (±δXi)	0.001	B	Uniform	-0.0927	0.0003	2.68E-05
Efficiency of extension line (η1)	1.000	none	Approximation (1σ)	0	-	-	-0.9268	0.0000	0.00E+00
			Resolution (±δXi)	0	B	Uniform	-0.9268	0.0000	0.00E+00
Efficiency of adaptor (ηa)	0.987	none	Approximation (1σ)	0.0007	A	t	-0.9368	0.0007	6.57E-04
			Resolution (±δXi)	0.0001	B	Uniform	-0.9388	0.0000	2.71E-05
Measured K	0.9151	none	Measurement (σn-1)	0.0005	A	t	1	0.0003	2.66E-04
Independent uncertainty			Γg	0.058	B	U	0.9268	0.0046	4.27E-03
			Γu	0.026					

Measurement equation $K = \frac{1}{C_d} \cdot \frac{P_u}{P_{mu}} \cdot \frac{1}{\eta_l \eta_a} \cdot \frac{1}{|1 - \Gamma_g \Gamma_u|^2}$ Combined standard uncertainty 0.0048

Table 4 shows the example of the uncertainty of K evaluated at 18 GHz based on the data above described and the combined standard uncertainty of the calibration came to 0.0048 which corresponds to 0.52% divided by its calibration factor.

To evaluate these C_d and K , a macro was made which

can automatically execute to the all frequency range. The complicated data processing became easily possible with this program in all frequency range.

Finally, **Fig. 2** shows the frequency characteristics of the calibration result with its uncertainty and the power splitting ratio C_d . Where, the uncertainty is

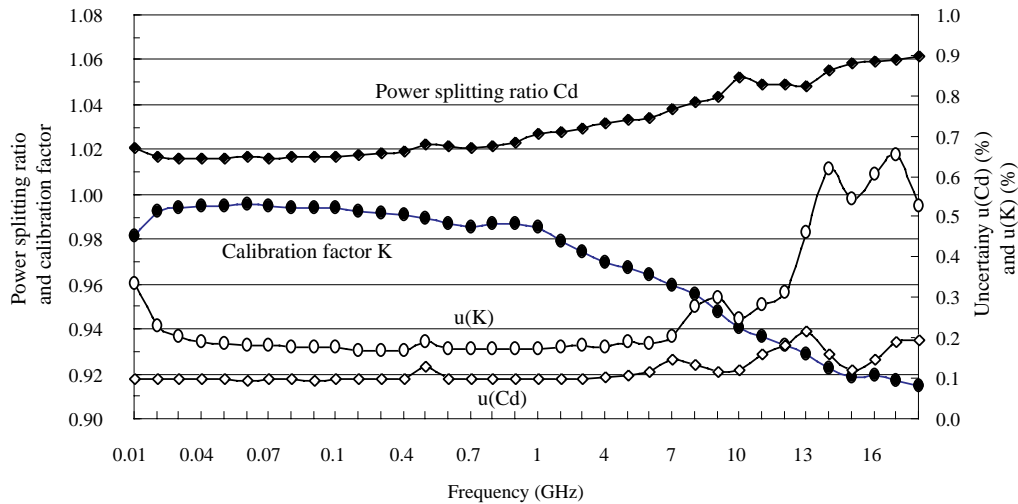


Fig. 2 Measured calibration factor and evaluated uncertainty

expressed in the relative standard uncertainty. $u(C_d)$ is the uncertainty of this calibration system itself and ranges between 0.1 and 0.21 %. For the DUT calibration, it became between 0.15 % and 0.65 % in the full frequency range. If the higher the reflection of a DUT, the larger the uncertainty becomes. The reason why the uncertainty becomes large in the low and high frequency is mainly due to the source mismatch.

§6 Conclusion

The evaluation of uncertainty of a broadband RF power calibration system was performed from 10 MHz to 18 GHz of a 7 mm coaxial waveguide. Uncertainty factors were summarized to three tables of the DC substitution coefficient, the power splitting ratio and the calibration factor. As a result, the relative standard uncertainty of the calibration system became 0.1~0.21 % for the frequency range from 10 MHz to 18 GHz. This is considered as the best uncertainty of the system. As an example, a power meter was calibrated and the uncertainty became 0.15~0.65 %. This system is scheduled to use for a broad band calibration of RF power meter with a coaxial waveguide.

We appreciate Mr. Kyouhei Yamamura of the Japanese Quality Assurance Organization who

cooperated with the development of this system.

References

- 1) T. Inoue: "Broadband power standard for coaxial waveguide in the frequency range of 10MHz-18GHz - Design and fabrication - " ETL bulletin this issue
- 2) T.Inoue and K.Yamamura: "A broadband power meter calibration system in the frequency range from 10 MHz to 40 GHz using a coaxial calorimeter", IEEE Trans. IM, **45**, 1, 146~152 (1996-2)
- 3) ISO: "Guide to the expression of uncertainty in measurement (1995)
- 4) T.Inoue: "Evaluation of excess heating by an adiabatic line in calorimetric measurement of RF power", CPEM 2000 (Sydney, Australia) to be presented(2000-5)

(Accepted January 31, 2000)