

## §3 Development of a TVC-Comparator

### 3.1 Introduction

In this chapter, a high-precision ac-dc difference comparator developed at ETL is described. The ac-dc difference comparator measures the relative ac-dc difference between two Thermal Voltage Converters (TVCs). The comparator is indispensable to build-up the chains of reference TVCs for different test-voltages.

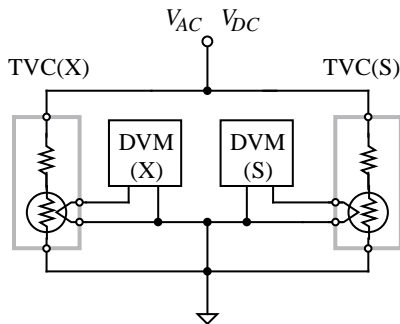
At the early stage of the development of the comparator at ETL, large systematic variations of measurement data of a few ppm have been observed. Hence new techniques have been employed in the comparator system to investigate the cause of the fluctuation, as will be described in the following sections.

### 3.2 Measurement principle

The new comparator system determines the relative ac-dc transfer differences of TVCs by two different simultaneous measurement[30], i.e., by a dual-channel method and a differential method. The results obtained by the two different methods are cross-examined to confirm the reliability of the measurement. The principles of the two methods are described in detail in the following sub-sections.

#### 3.2.1 Dual-channel method

In the dual-channel method, a measurement circuit developed by Yamazaki et.al.[35] has been used. The schematic diagram of the measurement circuit is shown in **figure 3.1**. In this method, the main detectors are the voltmeters DVM(X) and DVM(S) which measure the output EMF voltages of the two TVCs separately. This type of the dual-



**Figure 3.1** Schematic diagram of measurement circuit of the dual-channel measurement. The main detectors are the voltmeters DVM(X) and DVM(S) which measure the output EMF voltages of the two TVCs separately.

channel method it is also called the ‘Two-DVM method’ at ETL.

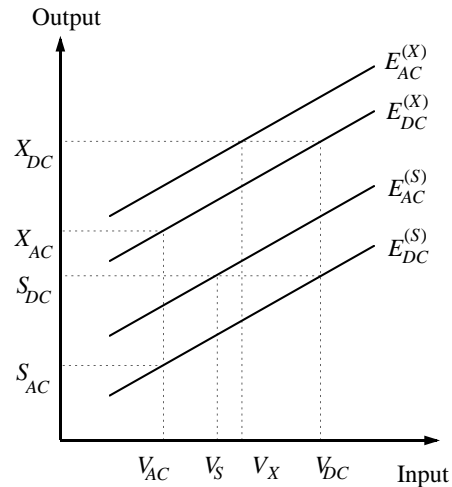
The input-output characteristics of the two TVCs are illustrated in **figure 3.2**. The EMF output of a TVC is approximately proportional to the square of the input voltage. The output-quantity  $X_{DC}$  and  $S_{DC}$  represent the EMF outputs from TVC(X) and TVC(S) for the dc input voltage  $V_{DC}$ . Similarly, the output-quantity  $X_{AC}$  and  $S_{AC}$  represent the EMF outputs for the ac input voltage  $V_{AC}$ . The input-quantity  $V_X$  and  $V_S$  represent the ac input voltages which produce the same EMF voltage ( $X_{DC}$ ,  $S_{DC}$ ) as in the case of applying the dc input voltage  $V_{DC}$ .

Using the definition of the ac-dc difference of a TVC given in (1.2), the relative ac-dc difference between TVC(X) and TVC(S) is deduced as

$$\delta_x - \delta_s \equiv \frac{V_x - V_s}{V_{DC}} \Big|_{\substack{X_{AC}=X_{DC} \\ S_{AC}=S_{DC}}} \quad (3.1)$$

If the difference between the dc input voltage  $V_{DC}$  and ac input voltage  $V_{AC}$  is small, the input-output characteristic of the two TVCs may be approximated to be linear in the small voltage range. In this case, the following approximation is possible:

$$\begin{cases} V_x \equiv V_{AC} + (X_{DC} - X_{AC})/k_x \\ V_s \equiv V_{AC} + (S_{DC} - S_{AC})/k_s \end{cases} \quad \text{where } k_x = \frac{\Delta X}{\Delta V}, k_s = \frac{\Delta S}{\Delta V} \quad (3.2)$$



**Figure 3.2** Input-output characteristics of two TVCs to be compared. The input-quantity  $V_X$  and  $V_S$  represent the ac input voltages which produce the same EMF voltage ( $X_{DC}$ ,  $S_{DC}$ ) as that for the dc input voltage  $V_{DC}$ .

Here,  $\Delta X$  and  $\Delta S$  represent the change in the EMF output from TVC(X) and TVC(S) when a small change in the input voltage  $\Delta V$  is applied. Substituting (3.2) to (3.1), the relative ac-dc difference  $\delta_x - \delta_s$  is determined by the following equation:

$$\delta_x - \delta_s \cong \frac{S_{AC} - S_{DC}}{n_s S_{DC}} - \frac{X_{AC} - X_{DC}}{n_x X_{DC}}$$

where,  $n_x = \frac{(\Delta X / X_{DC})}{(\Delta V / V_{DC})}$ ,  $n_s = \frac{(\Delta S / S_{DC})}{(\Delta V / V_{DC})}$ . (3.3)

The ‘normalized indices’  $n_x$  and  $n_s$  are of the order of 2 for the TVCs with square-output characteristics. The dual-channel method is convenient for the comparison of two TVCs with different input-output characteristic, such as the comparison between SJTC to MJTC or SJTC to Fluke 792A.

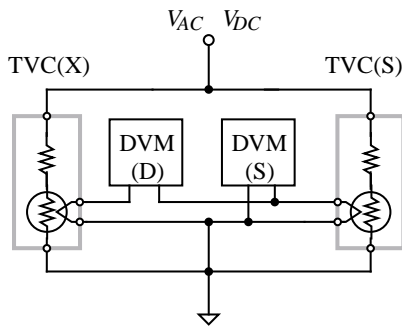
### 3.2.2 Differential method

The schematic diagram of measurement circuit of the differential method is shown in **figure 3.3**. In the differential measurement, the main detector is the digital voltmeter DVM(D) which detects the difference in the output EMF voltage of TVC(X) and TVC(S). Since the DVM measures only the difference in the EMF output of the two TVCs, the input-range of the DVM(D) may be set to the highest-resolution setting, while in the case of the two-DVM method, the DVMs have to take total EMF output from the two TVCs and loses one or more digits of resolution.

In differential measurement, the difference in the ac-dc difference between the TVC(X) and the TVC(S) is determined by an equation which is mathematically equivalent to (3.3):

$$\delta_x - \delta_s \cong -\frac{D_{AC} - D_{DC}}{n_x X_{DC}} - \frac{S_{AC} - S_{DC}}{n_x X_{DC}} \left(1 - \frac{\Delta X}{\Delta S}\right)$$

where,  $X_{AC} \equiv S_{AC} + D_{AC}$ ,  $X_{DC} \equiv S_{DC} + D_{DC}$ . (3.4)



**Figure 3.3** Schematic diagram of measurement circuit of the differential method. The main detector is the digital voltmeter DVM(D) which detects the difference in the output EMF voltage of TVC(X) and TVC(S). Since the DVM measures only the difference in the EMF output of the two TVCs, the input-range of the DVM(D) may be set to the highest-resolution setting.

The second term corrects for the difference of input-output characteristic of the two TVCs[30]. The correction term is calculated using the total EMF voltage from the two TVCs measured by the other two digital voltmeters DVM(X) and DVM(S).

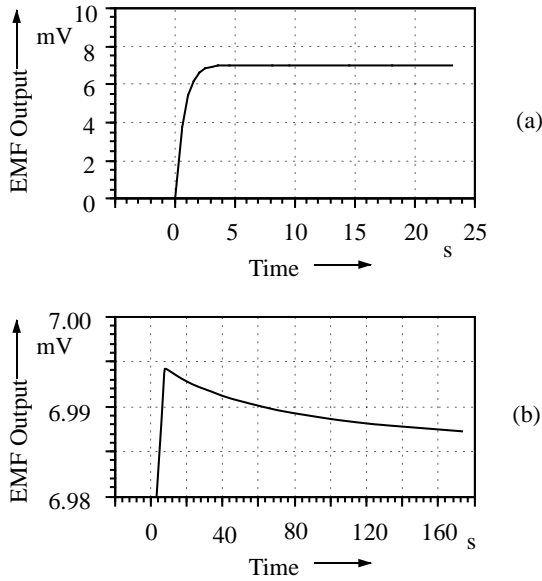
The second terms used to be neglected in the differential measurement in ETL, on the condition that the input ac voltage and the dc voltage are matched within  $10^{-5}$ . However, if the precision better than  $10^{-6}$  is required for the ac-dc transfer difference, the correction terms become more significant. Especially, when TVCs of different nominal voltage are compared, or when a SJTC is compared with a MJTC, the ratio of the indices  $n_x/n_s$  may differ from unity by more than 10%, and the correction term may contribute more than 0.1 ppm.

### 3.3 Effect of the off-time

At the early stage of development of the comparator system, a systematic variation of the ac-dc difference measurement as large as 1 ppm has been observed. The most significant variation always occurred for the first data of ten measurement sequences, and the variation was independent of the stabilization time before the initiation of the first sequence. Hence the variation was supposed to be related to some difference in mode switching between the first measurement sequence and the rest of the sequence.

At the mode switching between ac, DC+, and DC-, there are finite periods (‘off-time’) in which neither ac nor dc voltage is applied to the TCs. If a calibrator such as Fluke 5700A is used for the dc and ac source, the off-time is of the order of 1 second, and temperature of the heater decreases significantly during the off-time period. After the re-application of the input voltage, the temperature of the heater recovers to the steady-state as  $\exp(-t/\tau)$ , where  $\tau$  is the time-constant of the heater. In the case of SJTC which has a time-constant of about 1 s, a standard waiting-time of 20 s should be long enough for the output to recover to the original value within  $10^{-7}$ .

The cause of the variation has been traced to another much longer time-constant  $\tau'$  of the type SS283 SJTC, which is of the order of several minutes. When nominal current (10 mA) is applied to the SS283 SJTC, the response in the EMF-output is characterized by the two time constants ( $\tau$ ,  $\tau'$ ). An example of such joule heat-up is shown in the **figure 3.4**. After applying the current to the heater, the EMF output voltage reaches a constant value within a few seconds, as shown in figure 3.4(a). However, if the vertical axis is enlarged, a small exponentially decreasing drift of EMF is observed due to the second time constant, as shown in Fig. 3.4(b). The decreasing drift is typical for the SJTCs, and the decrease in the EMF voltage could be as large as 0.1% of the total EMF voltage. The cause of the second time constant  $\tau'$  is supposed to result from the heat-up of the whole TC element. In the case of SS283 SJTC which has relatively large temperature coefficient ( $\approx 0.1$  %/K) of the EMF output, the heat-up of the whole TC element by 1K can give rise to 0.1% de-

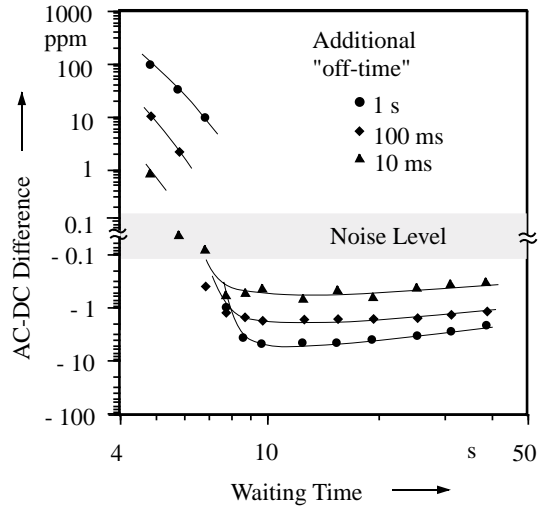


**Figure 3.4** Change in the output EMF of a type SS283 SJTC after nominal current (10 mA) is applied. The small exponentially decreasing drift, with a time constant of several minutes, is a typical characteristic of the SJTCs with Pt-Ir lead.

crease in the EMF voltage.

The effects of the off-time to the TCs were experimentally investigated using a linear solid-state thermal transfer standard (Fluke 792A) as the reference standard[20]. The rms sensor of Fluke 792A has very small thermal volume and hence a much faster response-time than SJTCs and MJTCs. The result for a SJTC (Best Technology model SS283), which is the same type as that used in the new standard TVCs, is shown in **figure 3.5**. In the measurement, additional off-time (10 ms, 100 ms and 1 s) were inserted in the mode-change from DC(-) to AC, and the change of the ac-dc difference has been measured. The horizontal axis is the waiting period before the DVM starts sampling the EMF output from the TCs.

Although the second time constant  $\tau'$  contributes only 0.1 % to the total EMF output, it does dominate the ac-dc transfer difference after a waiting-time longer than 10 s. As

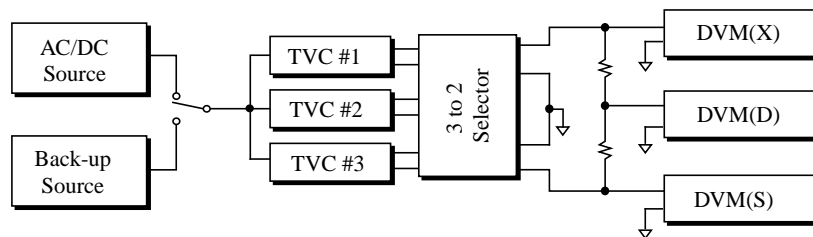


**Figure 3.5** The effect of ‘off-time’ in the mode-switching between ac and dc, in the case of a type-SS283. Additional off-time (10 ms, 100 ms and 1 s) were inserted during the mode-change from DC(-) to AC. The horizontal axis is the waiting period before the DVM starts sampling of the EMF output from the TCs.

shown in the figure, an off-time as short as 10 ms affected the measurement at 0.1-ppm level. Thus for the comparison of SJTCs, it is quite important to use fast switching elements to reduce the off-time below 10 ms.

### 3. 4 Measurement system

The ac-dc difference comparator is used to measure the difference in the ac-dc difference between two or three TCs. The most accurate measurement is required for the transfer of ac-dc difference from the MJTC of PTB to the new standard TVCs for different voltage ranges. The ac-dc difference comparator is also used in the calibration of the primary standard TVCs maintained at JEMIC. Hence in the development of the ac-dc difference comparator, the goal of the accuracy and the resolution of the measurement system was set to 1 ppm and 0.1 ppm, respectively. In this section, the construction (hardware) and the operation (software) of the



**Figure 3.6** A simplified diagram of the comparator system. In the differential measurement, the main detector is the digital voltmeter DVM(D), while in the dual-channel measurement, the main detectors are the voltmeters DVM(X) and DVM(S). A ‘back-up’ source is used in order to reduce the transient effect of the mode-switching between ac and dc. Two TVCs among the three are cyclically intercompared using a 3-2 selector.

ac-dc difference comparator are described in detail.

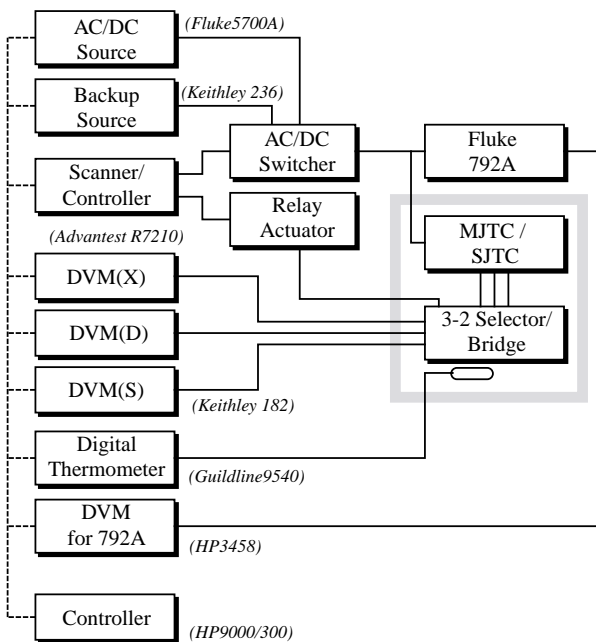
### 3.4.1 Construction

A simplified diagram of the new comparator system is shown in **figure 3.6**. As described in the section 3.2, the ac-dc transfer differences of TVCs are determined by the differential method and the dual-channel method simultaneously.

In the comparator system, two TVCs among the three are cyclically intercompared using a 3-2 selector in one cycle of measurements. Thus, if the three TVCs have ac-dc transfer difference of  $\delta_1$ ,  $\delta_2$ , and  $\delta_3$ , then three independent values [i.e.,  $(\delta_1-\delta_2)$ ,  $(\delta_2-\delta_3)$  and  $(\delta_3-\delta_1)$ ] are obtained in the measurement cycle. The consistency among the values is examined by a ‘zero-sum’ check at the end of the measurement.

The conventional method to obtain the difference in the EMF output from the TCs is to connect the output of the TVCs in series-opposite. In the comparator system developed at ETL, a bridge circuit is used for the differential measurement. Compared with conventional method, the signal to noise ratio is decreased to 1/2 in the bridge circuit, resulting in the increase of type-A uncertainty. The advantage of the bridge circuit is that all the ‘Input-Lo’ of the detectors can be connected to the ground potential. This configuration has better immunity to the external noises which may cause systematic errors in the measurement.

The construction of the comparator system is illustrated in **figure 3.7**. The measurement is completely automated using the HP9000/300 series controller. In order to mini-



**Figure 3.7** The schematic diagram of the comparator system. A commercially available calibrator (Fluke 5700A) is used for the voltage source of the system. The output EMFs from the TVCs are measured by the Keithley 182 nano-voltmeters.

mize the interference from external noises, the total measurement circuit is electrically isolated from the digital control circuit by an optical IEEE-488 bus and isolation transformers.

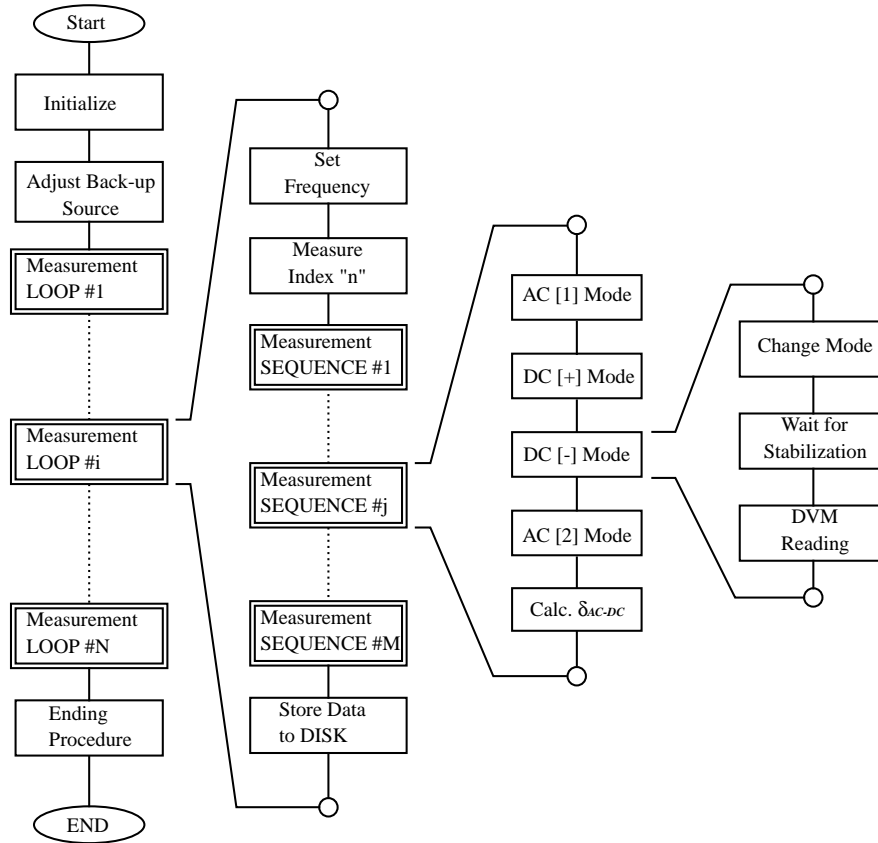
A commercially available calibrator (Fluke 5700A), which has a short term ac-output stability of better than 10 ppm, is used for the voltage source of the system. The calibrator requires nearly one second for switching from ac to dc modes, and in this period no voltage appears at the output of the calibrator. Hence the high-speed reed relays and a back-up source (Keithley 236 Source-measure unit) are employed for the purpose of reducing the effect of the ‘off-time’. In this configuration, the off-time is reduced to 1.3 ms which is equally distributed in the mode switching between ac and dc. While the TVCs are connected to the back-up source, a dummy resistance is connected to the calibrator in order to avoid the drift of the output due to the change in the load resistance.

In the comparison of the ac-dc difference in the higher frequency range (>10 kHz), frequency characteristic of a TVC including the connecting circuit between the TVCs becomes significant. Hence it is widely accepted to define the branch-point of a tee connector, which connects the TVCs and the ac/dc source, as the ‘reference-plane’ from which the frequency characteristic of a TVC is evaluated. In the case of standard TVC of the ETL which use type-N receptacle (N-R) as the input connector, the center of N-type tee connector (N-TA-RRR) is taken as the reference plane. In the case of a TVC with GR-connector at the input, the ac-dc difference of the TVC must be defined including a GR to N adapter.

Miniature-type low-thermal latching relays (Matsushita S2E-L2-5V) are used in the 3-2 selector. The short-term stability of the thermal EMF voltage produced from the latching relay were measured to be less than 0.1 nV, which has negligibly small effect in the ac-dc difference measurement. The 3-2 selector and the TVCs are placed in a 10 mm thick aluminum box to reduce the variation of room temperature. The temperature inside the box is monitored by the precision Pt-resistance thermometer during the measurement. The ‘Signal-Lo’ of the input circuit is one-point-connected to the aluminum box at the tee-connector. The aluminum box also serves as a shield for electromagnetic radiation noise.

The output EMFs from the TVCs are measured by three Keithley 182s. The low-noise triax cables are used for the wiring between the TVCs and Keithley 182s. The inner shield connects ‘Output-Lo’ of the 3-2 selector circuit to the ‘Input-Lo’ (circuit guard) of the Keithley 182. The outer shield of the cable is connected to the chassis ground of the Keithley 182 and the other side is left unconnected.

ETL is responsible for the ac-dc transfer standard in the voltage range 2 V to 20 V, and in the frequency range 40 Hz to 100 kHz. Hence the ac-dc comparator system is designed to operate in the voltage range 1 V to 100 V, and in the frequency range 10 Hz to 1 MHz.



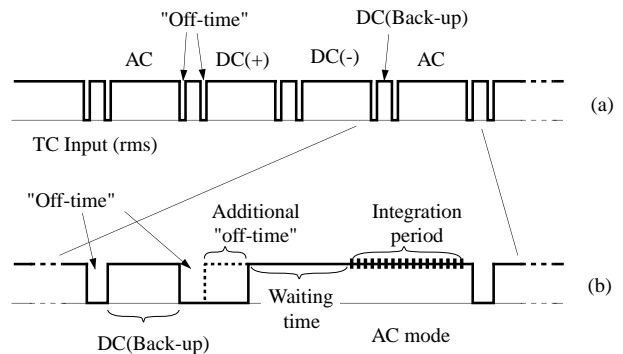
**Figure 3.8** A flow-chart of one set of measurements. Each measurement loop consists of a set of measurement sequence for a fixed frequency and a fixed voltage.

### 3. 4. 2 Measurement sequence

As described in section 3.2, the comparison of the ac-dc differences between the two TVCs (X and S) is performed using the ac-mode and two dc-modes (DC+, DC-), according to the equations (3.3) or (3.4). Since the EMF output from a SJTC at a rated voltage is of the order of several mV, it is very important to reduce the effect from the drift of the EMF output of TC or the drift of thermal EMFs in the output circuit. Hence, the standard measurement sequence (AC, DC+, DC-, AC) has been used for the purpose of canceling the effect of the drifts during the measurement.

A flow-chart of one set of measurements is shown in **figure 3.8**. After the initialization, the program tries to adjust the output of the ac source such that the output EMF from the TC equals to that of the dc mode within  $10^{-5}$ . Then the program normally waits 30 minutes for temperature stabilization before going to the measurement loops. Each measurement loop consists of a set of measurement sequence for a fixed frequency and a fixed voltage. In the measurement loop, the program measures the normalized index “n” of the TC by changing the output of the dc source by  $\pm 0.1\%$ . Then the program goes to the main measurement sequence (AC, DC+, DC-, AC). In each mode, the output EMF of the TC is measured by the Keithley 182 nano-voltmeter, and the data are averaged for a specified number of reading.

When the comparator system is used to compare the ac-



**Figure 3.9** Schematic diagram of measurement sequence (AC, DC+, DC-, AC). The ‘off-time’ of 1.3 millisecond is equally distributed between the modes. For examining the effect of the off-time at the mode-change, additional off-time are inserted at the mode-change from DC(-) to AC.

dc difference of two TVCs at 15 standard reference frequencies from 10 Hz to 1 MHz, it takes about one hour for each frequency point. Usually, the measurement is repeated twice in order to check the reproducibility of the data. In this case, whole measurement takes one and a half days.

A set of measurement sequence (AC, DC+, DC-, AC) is illustrated in **figure 3.9(a)**. The sequence is repeated ten times for each measurement point, and the average value and stan-

standard deviation are calculated from the ten measurements. After each mode-switching, the controller waits nominally 15 seconds to reject the output-transients from TVCs, and then take the average reading of DVM for about 30 seconds.

As described in the section 3.3, it is also quite important to use fast switching elements to reduce the ‘off-time’ below 10 ms, especially in the case of the comparison of SJTCs. In the present system, the switching is performed with an off-time of 1.3 ms which is equally distributed at the time of switching the output modes. When evaluating the effect of the off-time, the additional off-time (10 ms, 100 ms and 1 s) has been inserted only in the mode-change from DC(-) to AC, as shown in the figure 3.9(b).

### 3.4.3 Resolution

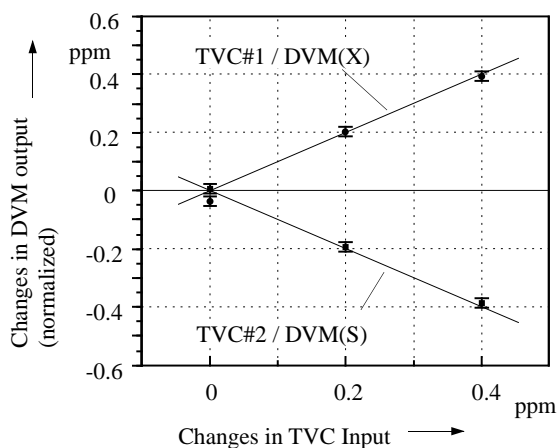
In the case of comparing standard TVCs at 60% of the nominal voltage, a typical output EMF voltage is of the order of 8 mV. A voltage resolution of 1.6 nV is required in order to obtain the resolution of 0.1 ppm in the ac-dc difference measurement, considering the input-output characteristic of the TVC:

$$\frac{dV}{V} = \frac{1}{n} \frac{dE}{E} \quad (3.5)$$

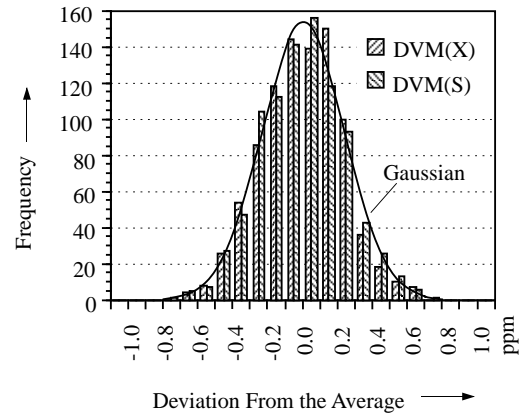
Hence the Keithley 182 nano-voltmeter, which has the highest display resolution of 1 nV at 3 mV range, is used in the comparator developed at ETL.

The actual voltage resolution of K182s is determined by the input-equivalent noise which was measured to be 14 nV/ $\sqrt{\text{Hz}}$ . Hence the integration time of 80 second is required in order to obtain the required voltage resolution of 1.6 nV.

While in the case of the standard TVC, the thermal noise from the output resistance (36 $\Omega$ ) is calculated to be 0.9 nV/ $\sqrt{\text{Hz}}$ . As a result, the resolution in the ac-dc difference measurement is mostly determined by the voltage resolution of K182s. Though the resolution of the K182 is satisfactory for



**Figure 3.10** Resolution of the comparator circuit measured experimentally by applying small changes  $\pm\Delta V$  around the rated dc voltage  $V_0$ . The error bars in the figure shows the experimentally obtained  $1\sigma$  uncertainties of the averages.



**Figure 3.11** Histogram of measured data for 300 measurement sequences. Distribution of the data is very close to the gaussian distribution.

the required precision, it is possible to improve the resolution by the use of other type of detectors optimized for low-resistance inputs.

The resolution of the detection circuit was examined experimentally. The voltage applied to the TCs was changed within a small voltage range ( $\pm\Delta V$ ) around the rated dc voltage  $V_0$ , and the changes in the outputs of TVC were measured by DVM(X) and DVM(S). The measurements were performed using a sequence ( $V_0 + \Delta V$ ,  $V_0 - \Delta V$ ,  $V_0 - \Delta V$ ,  $V_0 + \Delta V$ ) which was repeated 300 times to obtain one data-point.

The results of the measurements are shown in **figure 3.10**. The solid lines in the figure represent the response characteristic of the TVCs obtained by changing the input voltage as large as 0.1%. The output of the TVC#2 is oppositely connected to the input of DVM(S), and hence gives the negative slope in the figure. The error bars in the figure represents experimentally obtained  $1\sigma$  uncertainties for the average value of the 300 measurement sequence, which are evaluated to be 0.16 nV (0.014 ppm). Since the display resolution of the K182s in the 30 mV range is 10 nV, more than one decade of the improvement is obtained by the averaging. **Figure 3.11** shows the distribution of the data for the 300 measurements. As shown in the figure, the distribution of the data is very close to the gaussian distribution, as expected from the randomness in the nature of voltage-difference measurement.

### 3.5 Measurement uncertainty

The uncertainty in the calibration of a ‘‘test’’-TVC against the standard TVCs using the ac-dc difference comparator may be divided into the following three categories:

- The uncertainty in the evaluation of thermoelectric transfer difference of the reference TVC due to Thomson and Peltier effects.
- The uncertainty in the evaluation of low-frequency and high-frequency characteristic of the reference TVCs.

- (c) The uncertainty in the ac-dc difference comparison measurement using the comparator system developed at ETL.

The uncertainty of category (a) will be described in detail in section 8.3 for the FRDC method. The uncertainty of category (b) has been described in the sections 2.3 and 2.4. In this section, the uncertainty of category (c) will be described in detail. The uncertainty of category (c) is further divided into the following three groups, i.e., (c1) uncertainty due to TVC-input circuit, (c2) uncertainty due to TVC-output circuit, and (c3) uncertainty in index-measurement. Since these uncertainties are strongly dependent on the quality of the test-TVC, it is assumed that the test-TVC has a quality equivalent to the standard TVCs in the following evaluation.

### 3. 5. 1 Uncertainty due to TVC-input circuit

In this section, the possible causes of the uncertainty in the comparison measurement which are related to the input-circuit of the test-TVC and the reference TVC are evaluated.

#### (1) Frequency characteristic of the circuit

Since the chassis of the TVC is connected to the aluminum shield box via the 'Signal-Lo' of the input circuit, the effect of capacitance between the chassis and the aluminum box is negligible. The mutual inductance between the TVCs will not be a problem so long as the TVCs are set in a proper configuration to minimize the magnetic coupling.

The most significant effect is caused by the impedance of the input cable between the ac voltage source and the TVCs. Since the effect is strongly dependent on the system configuration, it is very important to evaluate the effect experimentally by changing the length of the input cable or by changing the earth-guard configuration.

#### (2) Stability of the input level

As expected from the equations (3.3) and (3.4), the fluctuations of the input level should not affect the ac-dc difference measurement because the fluctuation of output EMF voltages from the TVCs cancel each other. However, when different types of TVC are compared, the fluctuation of the input level may cause different fluctuation in the output EMF voltage due to difference in the time-constants. The effect is determined by the stability of the voltage source (Fluke 5700A) and may increase random error in the ac-dc difference measurement.

#### (3) DC offset

The dc-offset voltage in the ac-mode causes a first-order thermoelectric effect in the EMF output. Similarly if there is a difference in the amplitude of the DC+ and the DC- voltage, the first-order thermoelectric effect is not completely canceled. In the case of a SJTC which has reversal error of the order of 100 ppm, the dc-offset of 100 ppm causes a systematic error in the ac-dc difference measurement at 0.01 ppm level.

#### (4) Effect of off-time

In the present system, the input voltages are switched by high-voltage reed-relays which have an off-time of 1.3 ms. Since the off-time is equally distributed between the differ-

ent modes, and the reproducibility of the switching is within 0.1 ms, the effect of the off-time to the ac-dc difference measurement with the present system is expected to be smaller than 0.1 ppm.

#### (5) External noise

In the case of measuring 10V range SJTC at the rated input current of 10 mA, the total power of 0.1 W is applied to the TVC. Hence the noise may cause uncertainty in the measurement if the total noise power exceeds 0.1  $\mu$ W in the whole frequency range. Especially, if the noise is induced synchronously with the measurement sequence, the noise causes Type B uncertainty in the measurement.

The other possible source of errors related to the input circuit are as follows;

(a) Variation of the thermal EMF in the input circuit.

(b) Variation of the resistance of the range-resistor.

(c) Variation of the contact resistance of input-connector.

If these variations are slow enough and may be regarded as a linear drift with time, then the effect of these variations is compensated by the measurement sequence (AC, DC+, DC-, AC).

### 3. 5. 2 Uncertainty due to TVC-output circuit

In this section, the possible causes of the uncertainty in the comparison measurement which are related to the EMF-output circuit of the TVCs are evaluated.

#### (1) Output resistance of TC

In the case of the standard TVC, output resistance (9 $\Omega$ ) of four TC elements are connected in series. The total 36 $\Omega$  output resistance produces thermal noise of 0.7 nV/ $\sqrt{\text{Hz}}$ , which is much smaller than the input-equivalent noise of Keithley 182 nano-voltmeter (14 nV/ $\sqrt{\text{Hz}}$ ). On the other hand, due to the resistance of bridge circuit (1 k $\Omega$ ), mixing of the EMF output occurs between DVM(X) and DVM(S). The mixing is of the order of 2% and is corrected to less than 0.1 % using the measured output resistance at the time of measurement.

#### (2) Resolution of DVM

Though the display resolution of Keithley 182 nano-voltmeter is 1 nV at 3 mV input-range, the actual resolution is limited by the input-equivalent noise of the nano-voltmeter (14 nV/ $\sqrt{\text{Hz}}$ ). If distribution of the noise is gaussian, a reduction of the noise is possible by averaging the noise for certain period, as described in section 3.4.2. As a result, the two-DVM method which uses 30 mV input-range shows a similar detection sensitivity as in the case of the differential method which uses 3 mV range.

#### (3) Linearity of DVM

If the linearity of the DVM has local variation within 0.1 % of the range, as in the case of ladder-network D/A converter, the variation increases the uncertainty in the index-measurement. Also, if the reversal error of the TVC is larger than a few hundred ppm, the non-linearity of the DVM may increase the random error of the measurement. In the case of Keithley 182 nano-voltmeter, which uses pulse-width modu-

lation for the D/A circuit, the effect of the non-linearity is negligibly small so long as the input range does not change during one set of the measurement sequence.

#### (4) External noise

In the case of a measurement of TVC with single TC element, nominal output EMF is of the order of 7 mV. Hence the external noise with amplitude of the order of nV may affect the measurement. As in the case of external noise for the input circuit, the external noise may cause Type B uncertainty in case the noise is induced synchronously with the measurement sequence.

The other possible sources of errors related to the TC output circuit are as follows;

(a) Fluctuation of the thermal EMF in the TC output circuit.

(b) 'Pump-out' noise from the DVM

These sources of error may contribute to the type-A uncertainty which are strongly dependent on the measurement conditions, such as the stability of the ambient temperature. Hence it must be evaluated for each measurement point using the spread in the ac-dc difference value for the ten sets of measurement sequence. In the case of comparison between the standard TVCs, total type-A uncertainty ( $1\sigma$ ) for the mean ac-dc difference value for the 10 set of measurement sequence is 0.08 (0.14) ppm at 60(40)% of the rated input voltage.

### 3.5.3 Uncertainty in index-measurement

Since the index-measurements are performed only once for a group of measurement sequence, the uncertainty in the index-measurement must be treated as the source of systematic errors. The uncertainty in the index "n" contributes to the total uncertainty in the first order. The main source of error in the index-measurements is the resolution of the EMF voltages ( $\Delta X$ ,  $\Delta S$ ).

$$\begin{aligned} \frac{\partial n_x}{n_x} &= \frac{\partial(\Delta X)}{X_{DC}} \bigg/ \frac{\Delta X}{X_{DC}} \\ \frac{\partial n_s}{n_s} &= \frac{\partial(\Delta S)}{S_{DC}} \bigg/ \frac{\Delta S}{S_{DC}} \end{aligned} \quad (3.6)$$

The relationship of the uncertainty in the index measurement  $(\partial n)/n$  and the uncertainty contributed to the ac-dc difference comparison  $\partial(\delta_x - \delta_s)$  may be evaluated using (3.3):

$$\begin{aligned} [\partial(\delta_x - \delta_s)]^2 &\cong \left[ \frac{S_{AC} - S_{DC}}{n_s S_{DC}} \right]^2 \left[ \frac{\partial n_s}{n_s} \right]^2 \\ &+ \left[ \frac{X_{AC} - X_{DC}}{n_x X_{DC}} \right]^2 \left[ \frac{\partial n_x}{n_x} \right]^2 \end{aligned} \quad (3.7)$$

Assuming the worst cases in the resolution, the parameters  $\partial(\Delta X)/X_{DC}$ ,  $\partial(\Delta S)/S_{DC}$  are of the order of  $1 \times 10^{-6}$ . Combining the values  $\Delta X/X_{DC} \approx \Delta S/S_{DC} \approx 0.004$  for  $\pm 0.1\%$  change

in the input, the uncertainties in the index measurement  $(\partial n)/n$  are evaluated as  $2.5 \times 10^{-4}$ . In the ac-dc difference comparison measurement, the ac voltage is adjusted to the dc voltage such that  $X_{AC}$  should equal to  $X_{DC}$  within 10 ppm. Hence the uncertainty contributed to the ac-dc difference comparison measurement  $\partial(\delta_x - \delta_s)$  is estimated to be smaller than 0.01 ppm.

Non-linearity in the voltage source can also be a source of error in the index-measurement. Using (3.3), the uncertainty contributed to the ac-dc difference  $\partial(\delta_x - \delta_s)$  is evaluated to be:

$$\partial(\delta_x - \delta_s) \cong (\delta_x - \delta_s) \left[ \frac{\partial(\Delta V)}{V_{DC}} \bigg/ \frac{\Delta V}{V_{DC}} \right] \quad (3.8)$$

Since the output of dc voltage source (F5700A) is controlled by Pulse-Width-Modulation DAC circuit, its linearity  $\partial(\Delta V)/V_{DC}$  should not exceed more than 2 ppm. Using the values  $\Delta V/V_{DC} = 0.002$  for  $\pm 0.1\%$  change in the input, the uncertainty contributed to the ac-dc difference is estimated to be smaller than 0.1 % of the measured value  $\delta_x - \delta_s$ .

Another source of error may result from the second-order drift in the EMF voltage. Owing to the second time constants of the TC, the change of  $\pm 0.1\%$  in the input voltage may induce the second-order drift in the EMF. Since the second-order drift of the EMF is much smaller than 1 ppm during the index-measurement, the uncertainties in the index  $n$  are evaluated to be smaller than 0.1%. In either case, the contribution to the total uncertainty is evaluated to be smaller than  $10^{-8}$ , on condition that the matching of the ac voltage and the dc voltage are within  $10^{-5}$ .

## 3.6 Summary

For the purpose of improving the accuracy of the ac-dc transfer standards, a high-precision comparator has been developed at ETL. New techniques have been applied for improving the reliability of the comparator system; i.e., cyclic intercomparison using a 3-2 selector, and cross-checking of the data by two different simultaneous measurements. Possible systematic errors due to the transient effects of the switching between ac and dc modes have been investigated, and it was shown that an 'off-time' as short as 10 ms could affect the intercomparison at 0.1 ppm level due to the second time-constant of the TCs. By the use of 'back-up' source in order to reduce the effect of the off-time, the reproducibility in the measurement of relative ac-dc difference has been improved to 0.1 ppm level.

The new comparator has been used to calibrate TVCs by the use of the primary reference standards of ETL since 1994.