Appendix

A : Derivation of conductance matrix G

The conductance matrix G characterizes the frequency dependence of the TC circuit. In the calculation of the matrix, the U-shape wire-resistor is regarded as a distributed-parameter circuit consisting of small segments of length $\Delta l = l_0/N$, as shown in **figure A-1**. Applying the Kirchhoff's theorem to each of the segments, the following simultaneous equations are obtained.

$$
\begin{aligned}
& e_{x,n} = (1/j\omega\Delta C_s)\left[-\left(i_{x,n+1} - i_{x,n}\right) - i_{w,n}\right] \\
& e_{y,n} = (1/j\omega\Delta C_s)\left[-\left(i_{y,n+1} - i_{y,n}\right) + i_{w,n}\right] \\
& e_{x,n} - e_{x,n-1} = -(j\omega\Delta L + \Delta R)\cdot i_{x,n} \\
& e_{y,n} - e_{y,n-1} = -(j\omega\Delta L + \Delta R)\cdot i_{y,n} \\
& e_{x,n} - e_{y,n} = (1/j\omega\Delta C_w)\cdot i_{w,n}.
\end{aligned}
\qquad \text{(A.1)}
$$

Here, ΔC_s, ΔC_w represents capacitance to the chassis and capacitance between the wire, and ΔL_w, ΔR_w represent inductance and resistance. The parameters are defined per length Δl as,

$$
\begin{cases}
\Delta C_s = (\Delta l/l_0)C_s, & \Delta C_w = (\Delta l/l_0)C_w \\
\Delta L = (\Delta l/2l_0)L_w, & \Delta R = (\Delta l/2l_0)R_w .
\end{cases}
\qquad \text{(A.2)}
$$

Eliminating i_w in the equation (A.1), and using the notations as $\Delta e_{x,n} = e_{x,n} - e_{x,n-1}$, we obtain;

$$
\begin{aligned}
& e_{x,n} + e_{y,n} = -(1/j\omega\Delta C_s)\cdot\left(\Delta i_{x,n+1} + \Delta i_{y,n+1}\right) \\
& e_{x,n} - e_{y,n} = -(1/j\omega\Delta K)\cdot\left(\Delta i_{x,n+1} - \Delta i_{y,n+1}\right) \\
& i_{x,n} + i_{y,n} = -\left[1/(j\omega\Delta L + \Delta R)\right]\cdot\left(\Delta e_{x,n} + \Delta e_{y,n}\right) \\
& i_{x,n} - i_{y,n} = -\left[1/(j\omega\Delta L + \Delta R)\right]\cdot\left(\Delta e_{x,n} - \Delta e_{y,n}\right) \\
& \quad \text{where,} \\
& \qquad K \equiv 2C_w + C_s .
\end{aligned}
\qquad \text{(A.3)}
$$

Here, the new parameters e_+, e_-, i_+, i_- are defined as follows;

$$
\begin{cases}
e_+ = e_x + e_y, & e_- = e_x - e_y \\
i_+ = i_x + i_y, & i_- = i_x - i_y .
\end{cases}
\qquad \text{(A.4)}
$$

Using the new parameters and taking the limit $\Delta\lambda \to 0$, we obtain the following simultaneous differential equations.

$$
\begin{aligned}
& e_+ = \frac{-1}{j\omega C_{s0}}\cdot\left(\frac{di_+}{dl}\right) \\
& e_- = \frac{-1}{j\omega K_0}\cdot\left(\frac{di_-}{dl}\right) \\
& i_+ = \frac{-1}{j\omega L_0 + R_0}\cdot\left(\frac{de_+}{dl}\right) \\
& i_- = \frac{-1}{j\omega L_0 + R_0}\cdot\left(\frac{de_-}{dl}\right)
\end{aligned}
\qquad \text{(A.5)}
$$

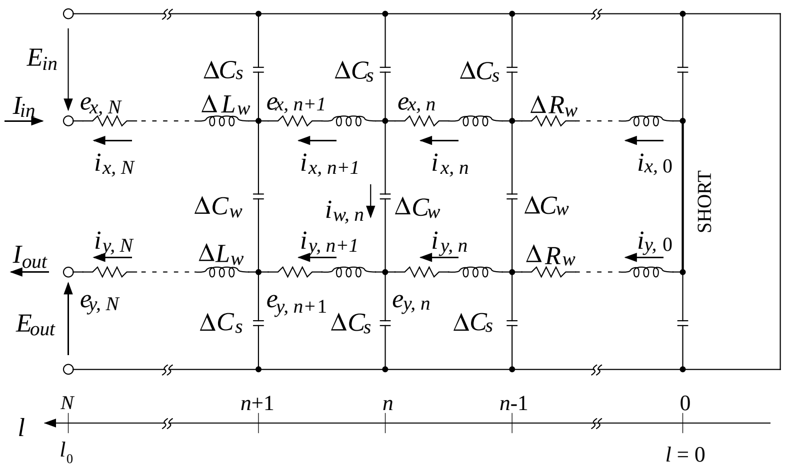

Figure A-1 *Equivalent circuit model of the input circuit of the HF-TVC. The U-shape wire-resistor is regarded as a distributed-parameter circuit, which consists of small segments of length* $\Delta l = l_0/N$. *The circuit parameters* ΔC_s, ΔC_w *represents capacitance from wire to the chassis and capacitance between the wire per length* Δl, *respectively.* ΔL_w, ΔR_w *represent inductance and resistance per length* Δl, *respectively.*

Here K_0, C_0, L_0, R_0 represents the circuit parameters per unit length, and are defined by,

$$\begin{cases} K_0 \equiv K/l_0, & C_0 \equiv C_s/l_0 \\ L_0 \equiv L_w/2l_0, & R_0 \equiv R_w/2l_0 . \end{cases} \quad \text{(A.6)}$$

Rearranging (A.5) for parameters e_+, e_-, we obtain,

$$\begin{cases} e_+ = \dfrac{1}{j\omega C_0 (j\omega L_0 + R_0)} \cdot \left(\dfrac{d^2 e_+}{dl^2} \right) \\ e_- = \dfrac{1}{j\omega K_0 (j\omega L_0 + R_0)} \cdot \left(\dfrac{d^2 e_-}{dl^2} \right) . \end{cases} \quad \text{(A.7)}$$

The (A.7) are the second-order linear differential equations, with general solution;

$$\begin{cases} e_+(l) = E_+ e^{\alpha l} + F_+ e^{-\alpha l} \\ e_-(l) = E_- e^{\beta l} + F_- e^{-\beta l} . \end{cases} \quad \text{(A.8)}$$

The parameters α, β have dimensions of the inverse of the length [L^{-1}], and are defined as follows;

$$\begin{cases} \alpha \equiv \sqrt{j\omega C_0 (j\omega L_0 + R_0)} \\ \beta \equiv \sqrt{j\omega K_0 (j\omega L_0 + R_0)} . \end{cases} \quad \text{(A.9)}$$

The boundary conditions for l=0 are,

$$\begin{cases} e_-(0) = e_x(0) - e_y(0) = 0 \\ i_+(0) = i_x(0) + i_y(0) = 0 . \end{cases} \quad \text{(A.10)}$$

Combining (A.10) to the general solution(A.8), we get,

$$\begin{cases} E_+ + F_+ = 0 \\ E_- - F_- = 0 . \end{cases} \quad \text{(A.11)}$$

The solution of the differential equation (A.7) under the boundary condition (A.10) are obtained as;

$$\begin{cases} e_+(l) = E_+ \cosh(\alpha l) \\ e_-(l) = E_- \sinh(\beta l) . \end{cases} \quad \text{(A.12)}$$

The solution for the parameters i_+, i_- are obtained by differentiating (A.12) and substituting them to (A.5);

$$\begin{cases} i_+(l) = -\dfrac{\alpha E_+}{j\omega L_0 + R_0} \sinh(\alpha l) \\ i_-(l) = -\dfrac{\beta E_-}{j\omega L_0 + R_0} \cosh(\beta l) . \end{cases} \quad \text{(A.13)}$$

On the other hand, the boundary conditions for $l=l_0$ are given by the following equations;

$$\begin{cases} e_+(l_0) = E_{in} + E_{out}, & i_+(l_0) = -I_{in} + I_{out} \\ e_-(l_0) = E_{in} - E_{out}, & i_-(l_0) = -I_{in} - I_{out} . \end{cases} \quad \text{(A.14)}$$

Substituting the parameters E_{in}, E_{out} in (A.14) by (A.12), the relationship between the input and output voltage of the four-terminal circuit is expressed as,

$$\begin{pmatrix} E_{in} \\ E_{out} \end{pmatrix} = \frac{1}{2} \cdot \begin{pmatrix} \cosh(\alpha l_0) & \sinh(\alpha l_0) \\ \cosh(\beta l_0) & -\sinh(\beta l_0) \end{pmatrix} \cdot \begin{pmatrix} E_+ \\ E_- \end{pmatrix} . \quad \text{(A.15)}$$

Similarly, substituting the parameters I_{in}, I_{out} in (A.14) by (A.13), the relationship between the input and output current is expressed as,

$$\begin{pmatrix} I_{in} \\ I_{out} \end{pmatrix} = \frac{1}{2(j\omega L_0 + R_0)} \cdot \begin{pmatrix} \sinh(\alpha l_0) & \cosh(\beta l_0) \\ -\sinh(\alpha l_0) & \cosh(\beta l_0) \end{pmatrix} \cdot \begin{pmatrix} \alpha & 0 \\ 0 & \beta \end{pmatrix} \cdot \begin{pmatrix} E_+ \\ E_- \end{pmatrix} . \quad \text{(A.16)}$$

Combining (A.15), (A.16), the conductance matrix for the four-terminal circuit is obtained as;

$$\begin{pmatrix} I_{in} \\ I_{out} \end{pmatrix} = \begin{pmatrix} A+B & A-B \\ -A+B & -A-B \end{pmatrix} \cdot \begin{pmatrix} E_{in} \\ E_{out} \end{pmatrix}$$

where

$$\begin{cases} A = \dfrac{\alpha}{2(j\omega L_0 + R_0)} \tanh(\alpha l_0) \\ B = \dfrac{\beta}{2(j\omega L_0 + R_0)} \coth(\beta l_0) . \end{cases} \quad \text{(A.17)}$$

B : Effect of off-time in FRDC-DC difference

In the case of modified FRDC waveform, the effect of the off-time needs to be evaluated. Using the Fourier transformation, the input current $i_{IN}(t)$ and the thermoelectric current $i_{TE}(t)$ are expressed as;

$$i_{IN}(t)/i_0 = \sum_{n=-\infty}^{+\infty} \alpha_n e^{in2\pi f_0 t}$$
$$i_{TC}(t)/i_0 = \sum_{n=-\infty}^{+\infty} \gamma_n e^{in2\pi f_0 t} . \quad \text{(B.1)}$$

The equation (6.10), which represents the response of the thermoelectric effect, can be expressed in a form of differential equation;

$$\frac{di_{TE}}{dt} = \frac{\delta_{TE} i_{IN} - i_{TE}}{\tau_{TE}} . \qquad \text{(B.2)}$$

The response function are readily obtained using (B.2) as,

$$\frac{\gamma_n}{\alpha_n} = 1 + \frac{\delta_{TE}}{1 - in2\pi f_0 \tau_{TE}} . \qquad \text{(B.3)}$$

Using the response function, difference in the total power between the CPDC- and MDFR-modes is calculated as,

$$\frac{P_{CPDC} - P_{MDFR}}{P_0} = \sum_{n=-\infty}^{+\infty} \left\{ |\gamma_n|^2_{CPDC} - |\gamma_n|^2_{MDFR} \right\}$$
$$\cong \sum_{n=-\infty}^{+\infty} \frac{2\delta_{TE}}{1 + (2\pi f_0 n \tau_{TE})^2} \left\{ |\alpha_n|^2_{CPDC} - |\alpha_n|^2_{MDFR} \right\} . \qquad \text{(B.4)}$$

Here, the higher order terms of δ_{TE} are neglected. The Fourier components for the CPDC- and MDFR-modes were calculated in section 6.3.1. Substituting the Fourier components in (B.4) by (6.29) and (6.30) we obtain the difference in the total power as,

$$\frac{P_{CPDC} - P_{MDFR}}{P_{CPDC}} = \frac{2\delta_{TE}}{1-\varepsilon} \left[(1 - 2\varepsilon) + \sum_{\substack{n=-\infty \\ n=even}}^{+\infty} \frac{1}{1 + (2n\pi f_0 \tau_{TE})^2} \left(\frac{2}{n\pi} \right)^2 \sin^2\left(\frac{n\pi\varepsilon}{2} \right) - \sum_{\substack{n=-\infty \\ n=odd}}^{+\infty} \frac{1}{1 + (2n\pi f_0 \tau_{TE})^2} \left(\frac{2}{n\pi} \right)^2 \cos^2\left(\frac{n\pi\varepsilon}{2} \right) \right]$$
$$\cong \frac{2\delta_{TE}}{1-\varepsilon} \left[\sum_{\substack{n=-\infty \\ n=odd}}^{+\infty} \frac{(2n\pi f_0 \tau_{TE})^2}{1 + (2n\pi f_0 \tau_{TE})^2} \left(\frac{2}{n\pi} \right)^2 - 2\varepsilon \right] . \qquad \text{(B.5)}$$

FRDC-DC difference is calculated using the formula (1.8), assuming that the output MF of the TC is proportional to the power P, as;

$$\delta_{FRDC-DC} = \delta_{TE} \left\{ (4 f_0 \tau_{TE}) \tanh\left(\frac{1}{4 f_0 \tau_{TE}} \right) - 2\varepsilon \right\} \Big/ (1 - \varepsilon)$$
$$= \delta_{TE} \left\{ \left(\frac{2\tau}{T_{SW}}\right) \tanh\left(\frac{T_{SW}}{2\tau} \right) - \frac{2 t_{off}}{T_{SW}} \right\} \Big/ \left(1 - \frac{t_{off}}{T_{SW}} \right) . \qquad \text{(B.6)}$$

Here, the following relation is used in the deduction of (B.6).

$$\frac{\tanh x}{x} = \frac{8}{\pi^2} \sum_{n=odd}^{+\infty} \frac{1}{n^2 + (2x/\pi)^2} . \qquad \text{(B.7)}$$

For most of the TCs, the time constant of the thermoelectric effect is more than two orders of magnitude bigger than the off-time (10μs). In this case, neglecting higher order terms of (t_{off}/T_{SW}), the (B.6) reduces to the formula (6.15) for the original waveform.

C : FRDC-DC difference due to TC-input circuit

In the case of original FRDC waveform, the change in the power-dissipation $\Delta P/P_0$ due to shunt capacitance is obtained by combining (6.26), (6.32) and (6.36) as,

$$\left[\frac{\Delta P}{P_0} \right]_{FRDC} = -1 + \sum_{\substack{n=-\infty \\ n=odd}}^{+\infty} \left(\frac{2}{n\pi} \right)^2 \frac{1 + (2n\pi f_0 \tau_r)^2}{1 + (2n\pi f_0 \tau_p)^2}$$
$$= \sum_{\substack{n=-\infty \\ n=odd}}^{+\infty} \left(\frac{2}{n\pi} \right)^2 \left[\frac{1 + (2n\pi f_0 \tau_r)^2}{1 + (2n\pi f_0 \tau_p)^2} - 1 \right]$$
$$= 16 f_0^2 \left(\tau_r^2 - \tau_p^2 \right) \sum_{\substack{n=-\infty \\ n=odd}}^{+\infty} \frac{1}{1 + (2n\pi f_0 \tau_p)^2} . \qquad \text{(C.1)}$$

Similarly the changes in the power $\Delta P/P_0$ for the “chopped” dc(CPDC) waveform and the modified fast-reversed dc(MDFR) waveform are calculated from (6.29) and (6.30):

$$\left[\frac{\Delta P}{P_0} \right]_{CPDC} = 16 f_0^2 \left[\tau_r^2 - \tau_p^2 \right] \sum_{n=-\infty}^{+\infty} \frac{\sigma_{even}}{1 + (2n\pi f_0 \tau_p)^2} \sin^2\left(\frac{n\pi\varepsilon}{2} \right)$$
$$\left[\frac{\Delta P}{P_0} \right]_{MDFR} = 16 f_0^2 \left[\tau_r^2 - \tau_p^2 \right] \sum_{n=-\infty}^{+\infty} \frac{\sigma_{odd}}{1 + (2n\pi f_0 \tau_p)^2} \cos^2\left(\frac{n\pi\varepsilon}{2} \right) . \qquad \text{(C.2)}$$

where σ_{odd} and σ_{even} are defined as

$$\sigma_{odd} = \begin{cases} 1 & (n = odd) \\ 0 & (n = even) \end{cases} , \quad \sigma_{even} = \begin{cases} 0 & (n = odd) \\ 1 & (n = even) \end{cases} . \qquad \text{(C.3)}$$

The relative difference for CPDC and MDFR is derived from (C.2):

$$\left[\frac{\Delta P}{P_0} \right]_{CPDC} - \left[\frac{\Delta P}{P_0} \right]_{MDFR} = 16 f_0^2 \left[\tau_r^2 - \tau_p^2 \right] \sum_{n=-\infty}^{+\infty} A_n \left[\sigma_{even} B_n - \sigma_{odd} C_n \right] \qquad \text{(C.4)}$$

where the terms A_n, B_n, and C_n are given by,

$$A_n = \frac{1}{1+(2n\pi f_0 \tau_p)^2}$$

$$B_n = \sin^2\left(\frac{n\pi\varepsilon}{2}\right), \quad C_n = \cos^2\left(\frac{n\pi\varepsilon}{2}\right). \tag{C.5}$$

The behavior of the terms A_n, B_n and C_n are illustrated in **figure C-1**. The term A_n has a Lorenz-type dependence to the parameter "*n*" with a half-width

$$n_a = 1/(2\pi C_p R_p f_0) = 1/(2\pi\tau_p f_0)\ . \tag{C.6}$$

The terms B_n and C_n have a repetition period

$$n_b = 1/(t_{off} f_0). \tag{C.7}$$

Using a mathematical formula

$$\sum_{n=-\infty}^{+\infty} \frac{\cos nx}{n^2+a^2} = \frac{\pi}{a}\frac{\cosh a(\pi - x)}{\sinh(a\pi)} \tag{C.8}$$

for the summation of the infinite series in (C.4), the change in the power-dissipation $\Delta P/P_0$ due to shunt capacitance is obtained as,

$$\left[\frac{\Delta P}{P_0}\right]_{CPDC} - \left[\frac{\Delta P}{P_0}\right]_{MDFR}$$

$$= 16 f_0^{\,2}\left(\tau_r^2 - \tau_p^2\right)\left[\sum_{\substack{n=-\infty \\ n=even}}^{+\infty} \frac{\sin^2(n\pi\varepsilon/2)}{1+(2n\pi f_0\tau_p)^2} - \sum_{\substack{n=-\infty \\ n=odd}}^{+\infty} \frac{\cos^2(n\pi\varepsilon/2)}{1+(2n\pi f_0\tau_p)^2}\right]$$

$$= 8 f_0^{\,2}\left(\tau_r^2 - \tau_p^2\right)\sum_{n=-\infty}^{+\infty} \frac{\cos(n\pi) - \cos(n\pi\varepsilon)}{1+(2n\pi f_0\tau_p)^2}$$

$$= 8 f_0 \tau_r \left(\frac{\tau_p}{\tau_r} - \frac{\tau_r}{\tau_p}\right)\left[\coth\left(\frac{1}{2 f_0 \tau_p}\right)\cosh\left(\frac{t_{off}}{\tau_p}\right)\right.$$

$$\left. - \sinh\left(\frac{t_{off}}{\tau_p}\right) - \mathrm{cosech}\left(\frac{1}{2 f_0\tau_p}\right)\right]. \tag{C.9}$$

Since the switching frequency f_0 is much larger than the characteristic time constants τ_p, the hyperbolic functions $\coth(1/2f_0\tau_p)$, $\mathrm{cosech}(1/2f_0\tau_p)$ can be approximated by 1 and 0, respectively. Then the influence of shunt capacitance in the input circuit on the FRDC-DC difference is calculated as

$$\left[\frac{\Delta P}{P_0}\right]_{CPDC} - \left[\frac{\Delta P}{P_0}\right]_{MDFR} \cong 8\tau_r f_0 \left(\frac{\tau_p}{\tau_r} - \frac{\tau_r}{\tau_p}\right)\exp\left(-\frac{t_{off}}{\tau_p}\right). \tag{C.10}$$

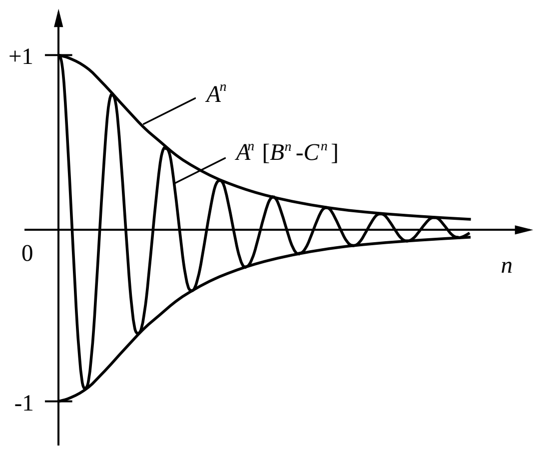

Figure C-1 *Frequency dependence of the terms A_n, B_n and C_n. The term A_n has a Lorenz-type dependence to the parameter "n" with a half-width $1/(2\pi\tau_p f_0)$. The terms B_n and C_n have a repetition period $1/(t_{off} f_0)$.*